

Introduction to Finance
Mid-term Exam, Correction of exercises 3 and 4 and of the problem

Exercise 3 (1.5 points)

You want to buy a house in Cergy to live in it next year, and inhabit it for 40 years. You can choose two ways of acquiring the house. The interest rate is 5%.

1. You can buy the house now for €450000
2. You can rent the house for 5 years for a cost of €2500 each month and then buy the house in year 6 at the same cost, €450000.

a. Compute the NPV of each option. Which one should you choose? (1 point)

We compute the present value of each option.

In case 1, this is simply a payment in period 0, no need to discount.

$$PV_1 = 450000$$

In case 2, I pay the rent, 2500 euros per month (I need to multiply by 12) during 5 years (annuity) and then buy the house in period 6.

$$PV_2 = \frac{2500 \cdot 12}{0.05} \cdot \left(1 - \frac{1}{(1.05)^5}\right) + \frac{450000}{(1.05)^6} = 465681$$

I prefer option 1 because $PV_1 < PV_2$.

b. You learn that there are property taxes of €5000 that need to be paid each year when you own the house. Does it affect your choice? Why? (0.5 point)

In case one the new PV_1 is: $PV_1 = 450000 + \frac{5000}{0.05} \left(1 - \frac{1}{(1.05)^{40}}\right) = 535595$. One has simply to add the taxes, which corresponds to an annuity.

$$\text{The new } PV_2 \text{ is } PV_2 = \frac{2500 \cdot 12}{0.05} \cdot \left(1 - \frac{1}{(1.05)^5}\right) + \frac{450000}{(1.05)^6} + \frac{5000}{0.05} \left(1 - \frac{1}{(1.05)^{35}}\right) \cdot \frac{1}{(1.05)^5} = 529829$$

In this case, taxes are only paid when you own the house, which is an annuity from the perspective of period 5, this is why we discount it by $\frac{1}{(1.05)^5}$.

I now prefer option 2 because its PV is smaller.

A. Problem (5 points)

You want to buy a company. The company is expected to generate, at the end of the year, a cash flow of 1.1 billion in year 1, 2 and 3. The third year an investment of €11.5 billion will

be necessary. Following this investment, the company will be able to generate, from period 4, €7.6 billion a year every year, for all time. Your discount rate is 10%.

- a. You negotiated a €40 billion deal to buy this company. Should you invest? (1 point)

The present value is: $-40 + \frac{1.1}{1.1} + \frac{1.1}{1.1^2} + \frac{1.1}{1.1^3} - \frac{11.5}{1.1^3} + \frac{7.6}{0.1} * \frac{1}{1.1^3} = 11.2$

The present value is positive so you should invest.

- a. This company has a negative impact on the environment. The government wants to tax the **positive cash flows** generated by the firm, so that investing in the company is less profitable than investing in a company developing green energy whose NPV is 1 billion. At what level should the tax be set? *Hint: The after-tax value of a cash-flow CF taxed at rate t is $CF*(1-t)$.* (1 point)

To know the level at which the tax should be set, one has to find the tax rate on the positive cash flows that will equalize the NPV of both investments. Note that the post-tax cash flow will be equal to $(1-t)*CF$, with t being the tax rate.

We have to solve (we just multiply the positive cash flows by $(1-t)$).

$$-40 + \frac{1.1(1-t)}{1.1} + \frac{1.1(1-t)}{1.1^2} + \frac{1.1(1-t)}{1.1^3} - \frac{11.5}{1.1^3} + \frac{7.6(1-t)}{0.1} * \frac{1}{1.1^3} = 1$$

We solve this equation for t.

$$(1-t) \left(\frac{1.1}{1.1} + \frac{1.1}{1.1^2} + \frac{1.1}{1.1^3} + \frac{7.6}{0.1} * \frac{1}{1.1^3} \right) = 1 + \frac{11.5}{1.1^3} + 40$$

$$(1-t)(59.84) = 49.64$$

$$(1-t) = \frac{49.64}{59.84} = 0.83$$

$$t = 1 - 0.83 = 0.17 = 17\%$$

A tax that is higher than 17% would make the other project preferable.

A first bank (bank 1) offers you a perpetual loan of €40 billion with growing repayments: next year, you will have to repay €1.7 billions, and the repayment will grow at 4.5% every year forever starting year 2.

- a. What is the NPV of the debt contract offered by bank 1? [*Hint: the NPV of a debt contract consists in a positive cash flow in period 0 and negative cash flows the following years*] (1 points)

The NPV of the debt follows the same logic as for other projects. Simply, there is a positive cash flow in period 0 and negative cash-flows in the future. The negative cash flows in the future have the structure of a growing perpetuity, so we can apply the formula. This is a debt project from the point of view of the individual that wants to buy a company, so we use its opportunity cost of capital:

$$NPV_1 = 40 - \frac{1.7}{0.1 - 0.045} = 9.1$$

Suppose now that you go to a second bank (bank 2). Bank 2 absolutely wants to be paid back in Year 10 exactly, starting next year. The repayment will also grow at 4.5% every year starting year 2.

- a. Which maximum repayment do you need to negotiate to choose bank 2's debt contract over bank 1's debt contract? (1 points)

We want to find C, the maximum repayment such that the second contract is preferred to the first one. We write the net present value of the second debt contract. We apply the formula of the growing annuity.

$$NPV_2 = 40 - \frac{C}{0.1 - 0.045} \left(1 - \frac{1.045^{10}}{1.1^{10}} \right)$$

Both NPV are equal if

$$40 - \frac{C}{0.1 - 0.045} \left(1 - \frac{1.045^{10}}{1.1^{10}} \right) = 40 - \frac{1.7}{0.1 - 0.045}$$

Which can be rewritten (subtracting 40 on each side, multiplying by -1, and multiplying by 0.1-0.045):

$$C \left(1 - \frac{1.045^{10}}{1.1^{10}} \right) = 1.7$$

Solving for C, we find that

$$C = \frac{1.7}{\left(1 - \frac{1.045^{10}}{1.1^{10}} \right)} = 4.24$$

The maximum acceptable initial repayment is 4.24 billion.

Unfortunately, you did not manage to negotiate that payment, so you went with bank 1 instead.

- a. What is the IRR associated with bank 1's debt contract? (0.5 point)

The IRR is the rate that equalizes the NPV to 0. We can simply take the NPV formula computed before.

Following this definition:

$$NPV(IRR) = 40 - \frac{1.7}{IRR - 0.045} = 0 \leftrightarrow IRR = \frac{1.7}{40} + 0.045 = 0.0875$$

The IRR of the first debt contract is 8,75%.

Suppose now that bank 1 cannot give the €40 billions as one loan. Instead, bank 1 must raise funds from multiple other lenders. It offers multiple smaller sized loans to choose from:

Debt Project	Loan Size	NPV
A	4	6
B	13	6.5
C	7	7
D	13	3.25
E	20	2

- a. Given that you have to raise **at least** \$40 billions, which debt projects are you going to pick? (0.5 point) *[Hint: Rank products by their profitability ratio.]*

Debt Project	Loan Size	NPV	Profitability Index
A	4	6	1.5
B	13	6.5	0.5
C	7	7	1
D	13	3.25	.25
E	20	2	0.1

We rank the project by decreasing profitability indexes and we pick them until we reach at least 40 billion. The ranking is: A, C, B, D, E

Projects A, C, B, and D allow to collect 37 billion. So we need to add project E to raise at least 40 billions.