Introduction to Finance

Sébastien Laffitte CY Cergy Paris University

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 - Compute the net present value of a project.

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 - NPV should be preferred to IRR or payback period when possible.
 - For mutually exclusive projects, pick the one with the highest NPV.
 - When resources are constrained, chose the projects that maximize total NPV.
 - \longrightarrow When you can found several different projects, compute the profitability index ($\frac{NPV}{Investment}$) to chose the most profitable ones.

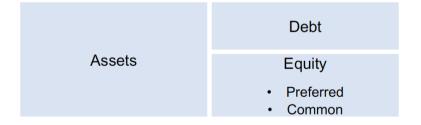
Today

- In previous sessions we have used the *r*, the discount rate as given.
 - Easy to compute for riskless projects: it corresponds to the risk-free rate (e.g. U.S. government bonds).
 - More complicated for risky projects.
- In this session we will learn how to compute it.

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- Don't forget: midterm net week.
 - Multiple-choice questions and exercises as in the problem sets.
 - 1h30 max.

- The Capital Structure of a firm is a mix of liabilities to finance assets.



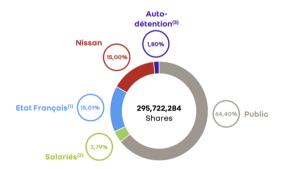
How are common stocks traded

- A company like Renault has around 300 millions shares.

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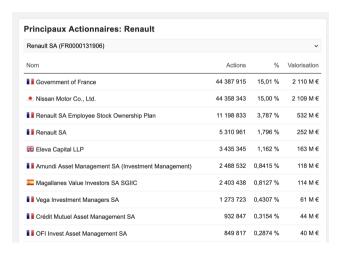
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Répartition au 31 décembre 2022



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Exercice	Nombre de titres composant le capital social au 31 décembre	Dividende par action (en euros)	
2015	295 722 284	2,40	
2016	295 722 284	3,15	
2017	295 722 284	3,55	
2018	295 722 284	3,55	
2018	295 722 284	3,55	

How are common stocks traded

- If Renault wants to raise new capital, it can borrow or or sell new shares to investors on the primary market.
- For now, most of action happens on the secondary market.



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	06-10	07-10	08-10	09-10	10-10
DER.	35,200	36,160	35,500	35,150	34,330
VAR.	-1,62%	+2,73%	-1,83%	-0,99%	-2,33%
OUV.	35,700	35,680	35,250	35,540	35,280
+HAUT	35,720	36,410	35,980	36,110	35,490
+BAS	34,330	35,470	35,170	35,150	34,330
VOL.	1508 707	1 411 183	1 086 081	880 527	1027 936

How are common stocks valued

- How to value these stocks?
- One can look at the quarterly balance sheet: in 2024Q4, the assets of Renault were 129bn euros.
- Renault's Equity was 31bn: this is the **book value** of Renault's equity.
- Does it represent the value of Renault?

How are common stocks valued

- How to value these stocks?
- One can look at the quarterly balance sheet: in 2024Q4, the assets of Renault were 129bn euros.
- Renault's Equity was 31bn: this is the **book value** of Renault's equity.
- Does it represent the value of Renault?
 - Not really because these assets are valued on a historical basis.
 - Renault market capitalization is 10bn!

How are common stocks valued

- One can also look at the liquidation value: what business would immediately receive upon selling the asset on the open market.
 - Depreciated assets might still have a value on the market
 - Some assets (e.g. real estate) might have appreciated.
 - Useful for creditors but... does not take intangible assets into account for instance.

- Just as with the PV of an asset or a project, we use the Discounted Cash Flow (DCF) formula to value stocks.
- Cash flows are discounted by the return that can be earned in the capital market on securities of a comparable risk.

$$\longrightarrow$$
 $PV(stock) = PV(expected future dividends)$

- A stock brings cash through:
 - Dividend payment (DIV_1)
 - Capital gain: $P_1 P_0$
- The expected rate of return on a stock is then:

$$r = rac{ egin{array}{c} ext{Dividend} & ext{Capital gain} \ ext{DIV}_1 & + egin{array}{c} ext{P}_1 - ext{P}_0 \ ext{Dividend} & ext{Polynomial price} \end{array}$$

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$$r = \frac{DIV_1 + P_1 - P_0}{P_0}$$

- *r* is the cost of equity capital: the expected return on other securities with the same risk.
- If a company offers a lower return than stocks of similar risk, investors sell stock because they can get a better return for no additional risk: $P_0 \searrow \implies r \nearrow$
 - To check this just compute $\frac{\partial r}{\partial P_0}$
 - The equilibrium price makes investors indifferent between this stock and other similar investments.

- Expected dividend next year: $D_1 = 10$
- Required return for similar risk: r = 10%

$$P_0 = \frac{10}{1.10} = 9.09$$

The determinants of stock prices

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Interpretation:

- If the stock trades at €9.09, expected return = 10%.
- If price > €9.09, return < 10% \rightarrow investors sell, P_0 decreases.
- If price < €9.09, return > 10% → investors buy, P_0 increases.

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Equilibrium price = €9.09.

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- We can then rewrite P_0 :

$$P_0 = \frac{1}{1+r} \left(DIV_1 + \frac{DIV_2 + P_2}{1+r} \right) = \frac{DIV_1}{1+r} + \frac{DIV_2 + P_2}{(1+r)^2}$$

- Investors might value the company today because they expect dividends in the future.

The determinants of stock prices

- Keeping this logic in mind for all the next periods, we get the general stock price formula or DCF formula:

$$P_0 = \sum_{t=1}^{\infty} \frac{DIV_t}{(1+r)^t}$$

- Despite being a source of cash, capital gains disappear from the formula.
- The share value is equal to the discounted stream of dividends per share.

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The cost of equity capital

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- The expected return is the dividend yield + the expected rate of growth.

- In the U.S. the price charged by electric and gas utilities are regulated.
 - Keep consumer price low but have ensure the utilities earn their fair rate of return.
 - The fair rate is precisely proxied by *r*: the cost of equity, or the rate offered by securities that have the same risk as the utility's common stock.

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- Utilities are mature and stable companies for which the constant-growth DCF formula is expected to work well.

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$$-r = \frac{DIV_1}{P_0} + g = 0.04 + 0.061 = 0.101$$

An application

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$$-r = \frac{DIV_1}{P_0} + g = 0.04 + 0.061 = 0.101$$

- Its cost of capital is 10.1% according to this method.

- Alternatively we can use the payout ratio that is the ratio of dividends to earning per share (EPS).
- Company A's payout ratio is expected to be 60%: each year the company pays 60% of its earnings in dividends and plows back 40% of it.
- Plowback ratio = 1- payout ratio = 0.4
- Company A's ROE is 11%.
 - Note that $ROE = \frac{EPS}{book \ equity \ per \ share}$
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- By how much book equity increases?
 - $-0.4 \times 0.11 = 0.044$
 - Earnings and dividends will also increase by 4.4%.
- Therefore, $g = plowback \ ratio \times ROE$ is an alternative formula.

Limits of the DCF formula

- Regular future growth is at best an approximation that can only be computed with errors.
 - Compute it for many similar companies and take an average to get a more reliable benchmark.
- Estimation with varying growth rates is possible: multi-stage DCF.
 - Young firms grow fast but it eventually stabilizes at lower rates (growth stocks vs. income stocks).
 - Decompose the DCF formula e.g. if dividend growth stabilizes after 3 years:

$$-P_0 = \frac{DIV_1}{1+r} + \frac{DIV_2}{(1+r)^2} + \frac{DIV_3 + P_3}{(1+r)^3} = \frac{DIV_1}{1+r} + \frac{DIV_2}{(1+r)^2} + \frac{DIV_3}{(1+r)^3} + \frac{1}{(1+r)^3} \times \frac{DIV_4}{r-g}$$

- Company A has $DIV_1 = 0.5$ and $P_0 = 50$. Its book equity at year 1 is 10\$.
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 - $r = \frac{.5}{50} + 0.2 = 0.21$
 - The assumption that a firm can grow by 20% per year is too strong.
- From year 3, ROE = 16% and plowback ratio = 50%. It means that g drops to 0.5*0.16 = 0.08.
- Compute *r*.

	Year				
	1	2	3	4	
Book equity	10.00				
Earnings per share, EPS					
Return on equity, ROE	.25	.25	.16	.16	
Payout ratio	.20	.20	.50	.50	
Dividends per share, DIV	.50				
Growth rate of dividends (%)	_				

	Year				
	1	2	3	4	
Book equity	10.00	12.00	14.40	15.55	
Earnings per share, EPS	2.50	3.00	2.30	2.49	
Return on equity, ROE	.25	.25	.16	.16	
Payout ratio	.20	.20	.50	.50	
Dividends per share, DIV	.50	.60	1.15	1.24	
Growth rate of dividends (%)	_	20	92	8	

Exercise

- We can use the multi-stage DCF formula.

$$P_0 = \frac{DIV_1}{1+r} + \frac{DIV_2}{(1+r)^2} + \frac{DIV_3 + P_3}{(1+r)^3} = \frac{DIV_1}{1+r} + \frac{DIV_2}{(1+r)^2} + \frac{DIV_3}{(1+r)^3} + \frac{1}{(1+r)^3} \times \frac{DIV_4}{r-g}$$

In the case of our exercise:

$$P_0 = \frac{0.5}{1+r} + \frac{0.6}{(1+r)^2} + \frac{1.15}{(1+r)^3} + \frac{1}{(1+r)^3} \times \frac{1.24}{r - 0.08}$$

- Trial and error gives $r \approx 0.099$

- So far, we have used a single discount rate r to value cash flows.
- But a firm is financed by **both equity and debt**.
 - Equity holders require a return r_E (reflecting higher risk).
 - Debt holders require a lower return r_D (since debt is safer and interest is tax-deductible).
- The overall cost of financing the firm's assets is a **weighted average** of these two sources of capital.
- This overall rate is called the Weighted Average Cost of Capital (WACC).
- It represents the opportunity cost of investing capital in the firm rather than elsewhere with similar risk.

The Weighted Average Cost of Capital (WACC)

- Given the capital structure, what is its cost?

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$$r_{WACC} = \frac{E}{E+D}r_E + \frac{D}{E+D}(1-T)r_D$$

- E for equity and D for debt.
- r_F the cost of equity.
- r_D the cost of debt.
- E + D = Assets. $\frac{E}{E+D}$ and $\frac{D}{E+D}$ the financing weights.
- T is the corporate tax rate.
- For an unlevered firm (i.e with no debt): $r_{WACC} = r_{E}$

The Weighted Average Cost of Capital (WACC)

- If equity is split between common equity and preferred equity, then

$$r_{WACC} = \frac{C}{C + P + D} r_C + \frac{P}{C + P + D} r_P + \frac{D}{C + P + D} (1 - T) r_D$$

- Preferred equity: hybrid between debt and equity. Preferred equity is more senior *i.e.* it has priority in the payment of dividends.
- It is generally associated with fixed dividends and comes without voting rights.

The Weighted Average Cost of Capital (WACC)

- How should we compute financing weights?

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 - Book values: reflect accounting and historical quantities.
 - Market values: forward-looking measures based on how the firm is valued by investors.
- When the firm is tradeable and the market values observable, then use **market values** (this is what investors care about).

- A company has:
 - Debt with a face value of 10m, trading at 95%
 - Book equity of 10m.
 - 1 million shares of common stock tradeing at 30\$ per share.
 - What are the financing weights?

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-
$$MVD = 0.95 \times 10 = 9.5m$$

-
$$MVE = 30 * 1m = 30m$$

- MV = 39.5m
- Financing weights are therefore: 24.1 for debt and 75.9 for equity.

- Central to the WACC are r_E and r_D .
- These are the cost of equity and debt at which current investors agreed to finance the firm.
- To do so, they had to forego other investment opportunities with similar risk.
- r_E and r_D already incorporate the compensation to investors for foregoing these alternative opportunities.
 - If it was not the case they wouldn't invest and r_E and r_D would increase to attract more investors and compensate them adequately.
 - If they were overestimated, more investors would come in and they would decrease.
- r_{WACC} is therefore the r we are after: the opportunity cost of capital.

The Cost of Debt

- The firm's cost of debt r_D is the opportunity cost for an investor of holding debt from the firm.
- This rate must be set so that, given its price, creditors want to keep holding the firm's debt.
- r_D is such that: $P_{debt} = NPV_{debt}(r_D)$.
- The rate that equates the PV of the bond's future payment to today's market price is the **yield-to-maturity**.
 - This is the discount rate that sets the PV of the promised bond payments equal to the current market price of the bond
 - This is the rate an investor would earn by buying the bond at today's price and holding it to maturity.
- $r_D = YTM$

The Cost of Debt

- Imagine an OAT that has a face value of 100, pays a 8.5% rate and has a market price of 120.44.
- The NPV is:

$$\frac{8.5}{1+y} + \frac{8.5}{(1+y)^2} + \frac{8.5}{(1+y)^3} + \frac{108.5}{(1+y)^4}$$

- The price is 120.44, so the YTD is defined as the rate for which:

$$P = NPV \Leftrightarrow 120.44 = \frac{8.5}{1+y} + \frac{8.5}{(1+y)^2} + \frac{8.5}{(1+y)^3} + \frac{108.5}{(1+y)^4}$$

- The YTM is 3% here. You can find it by trial and error.

The Cost of Debt

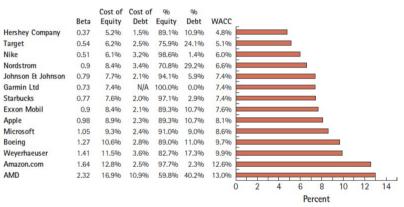
- The cost of debt vis-a-vis investors is the YTM but this is not the actual cost of issuing debt.
- Tax shield of debt = interest expenses that are deductible for tax purposes = interest expenses × tax rate.
 - Tax expenses = (EBIT Interest expenses) \times Tax rate
- The firm pays interest to creditors but receive a tax break in return.
- Therefore the effective cost of debt for the firm is $r_D^* = r_D(1 T)$

The Cost of Equity

The Cost of Equity

- r_E is simply the opportunity cost of holding equity from the firm.
- This is precisely what we computed at the beginning of the course.
- We will see in a few classes an alternative way of computing it.

WACC



2016 numbers

Capital Budgeting Process

1. Given the capital structure find the appropriate discount rate of the firm using WACC

$$r_{WACC} = rac{E}{E+D}r_E + rac{D}{E+D}(1-T)r_D$$

2. Compute the NPV of the project

$$NPV = \sum_{t=0}^{T} \frac{CF_t}{(1 + r_{WACC})^t}$$

3. Make investment decision using the appropriate investment decision rule.