Introduction to Finance

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Last week

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- The objective of the financial manager is to maximize shareholder value.
- Analysis of Financial Accounts.
  - Ratio analysis.

Today

- Introduction to financial mathematics.
- Future Value, Present Value and Yield.
- Single payment securities and multiple payment securities.

The time value of money

- A dollar today is worth more than a dollar in the future.
  - Individuals prefer it.
  - Inflation.
  - Uncertainty about the future.
- Finance is about valuing assets but for this we need to compare them adequately.
- We cannot compare values at different point in time.
  - How much 1$ in one year is worth today?

The time value of money

- **Discounting**: Bring future CF in today's value
- **Compounding**: Bring today's CF into the future.
- The discount rate makes the link between each period. A higher discount rate means a higher preference for the present and then a lower value of the CF in the future.
- Present value principles:
  - CF at different points in time cannot be aggregated. You need to bring them to the same point in time.
  - A good investment depends on how much you can get a CF but also when you can.

The different types of Cash-Flows

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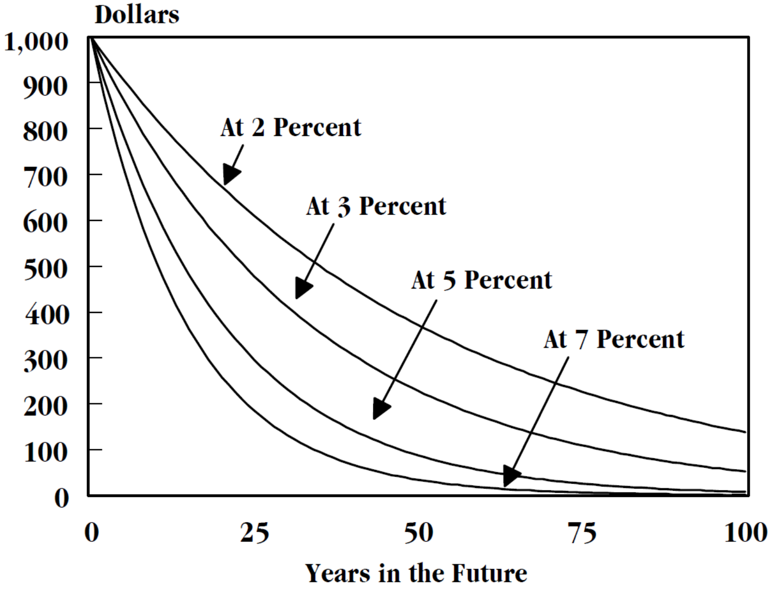
The different types of Cash-Flows

- Most assets are a combination of:
  - Simple CF: single CF in the future.
  - Annuities: constant CF that occur at regular intervals (not necessarily year).
  - Growing annuity: a CF that grows over time (ex. the CF increases by 5% each year).
  - Perpetuity: Constant CF at regular intervals for the rest of time.
  - Growing perpetuity: Growing CF for the rest of time.


Dollars
1,000
900
800
700
600
500
400
300
200
100
0
At 2 Percent
At 3 Percent
At 5 Percent
At 7 Percent
0
25
50
75
100
Years in the Future

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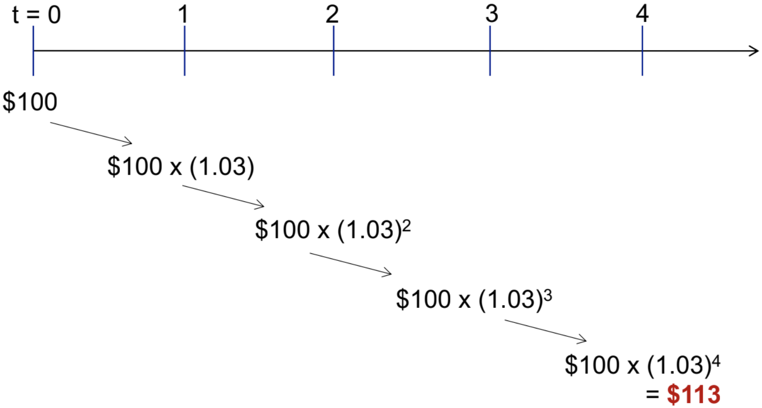
- $FV = PV \times (1 + r)^T = 1000 \times (1.05)^2 = 1102.5$
- Can be decomposed as:

$$1102.5 = \underbrace{1000}_{Principal} + \underbrace{2 \times 0.05 \times 1000}_{Simple\ interests} + \underbrace{0.05 \times 0.05 \times 1000}_{Interests\ on\ interests}$$

Future Value



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An example

- In 1626, Peter Minuit bought Manhattan from the Canarsie Indians for 24 dollars' worth of beads and trinkets. New Yorkers often call this the last real estate bargain in New York.
- The value of Manhattan is now estimated at $1.74tn.
- Was it a good deal?

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  - Always think in terms of opportunity costs.
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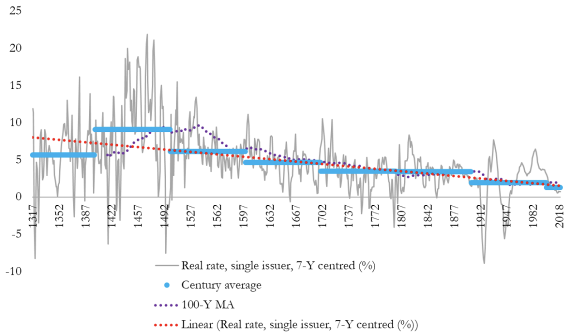
  - Always think in terms of opportunity costs.
- What is the rate of return? Let's compute the yield r.
- $PV = \frac{FV}{(1+r)^t} \Leftrightarrow (1+r) = \left(\frac{FV}{PV}\right)^{1/t}$
- $(1+r) = \frac{1.74 \times 10^{12}}{24}^{\frac{1}{2023-1626}} = 1.065$

An example

- The rate of return of Manhattan island is 6.5%.

- What is the return on a safe asset?

An example



Centennial averages

%	1300s	1400s	1500s	1600s	1700s	1800s	1900s	2000s
Nominal rate	7.3	11.2	7.8	5.4	4.1	3.5	5.0	3.5
Inflation	2.2	2.1	1.7	0.8	0.6	0.0	3.1	2.2
Real rate	5.1	9.1	6.1	4.6	3.5	3.4	2.0	1.3

Figure VII: Real long-term "safe asset" rates and composition by century, 1311-2018.[23]

Source: Schmelzing, 2022

An example

- The rate of return of Manhattan island is 6.5%.
- The long-term return on safe-assets is $\approx$ 4.3%.
- What's the future value of $24 invested on the safe asset in 1626?
- $FV = 24 \times (1.043)^{397} = 4.356 \times 10^8$
- It was a very good deal!

Formulas

- $FV = PV(1+r)^t$
- $PV = \frac{FV}{(1+r)^t}$
- $(1+r) = \left(\frac{FV}{PV}\right)^{1/t}$
- $t = \frac{log(\frac{FV}{PV})}{log(1+r)}$

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Pricing securities: Single payment security (Zero coupon bond)

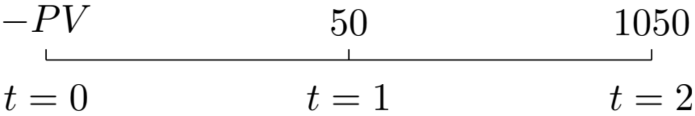
- The simplest fixed-income asset.
  - No interest payment.
  - At maturity, the owner is paid the par value of the bond (C)
  - Bond bought below the par value.
- The price is: $PV = \frac{C}{(1+r)^T}$
- Lower interest rates mean higher bond prices.

Pricing securities: Multiple payment security

- Can be seen as a collection of zero coupon bonds.
- PV of CF $C_0, C_1, \ldots, C_T$:

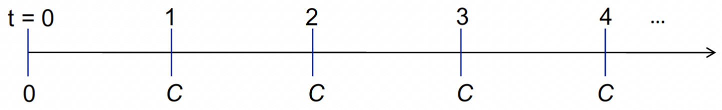
$$PV = C_0 + \frac{C_1}{(1+r_1)^1} + \ldots + \frac{C_t}{(1+r_t)^t} + \ldots + \frac{C_T}{(1+r_T)^T}$$

- What's the PV of this bond at a 5% interest rate?



Pricing securities: Perpetuities

- A perpetuity is a bond that pays a fixed payment C forever. No principal payment.
- Used in the 13th century between individuals or by States to raise capital.
- Existed in the U.K. and the U.S. more recently (see *consols*).
- Perpetuities help to think about stocks which are infinite (residual) claims on corporation profits.



Pricing securities: Perpetuities

- $PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + ... = \frac{C}{r}$
- Note that the price decreases when then yield increases.
- Suppose your cell phone plan costs $20 \times 12 = 240$ euros a year. Assume the interest rate at all maturities is 5%.
- What price are you willing to pay for a ”forever” plan?

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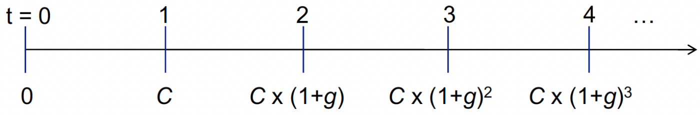
- $PV = \frac{C}{r} = \frac{240}{0.05} = 4800.$

Pricing securities: Growing Perpetuities

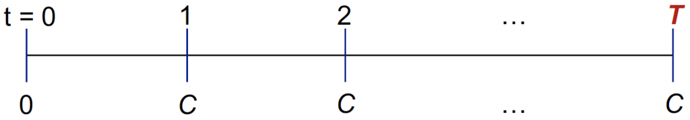
- A growing perpetuity is a stream of cash flows that occur at regular intervals and grow at a constant rate, g, forever



- $PV = \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \ldots = \frac{C}{r-g}$

Pricing securities: Annuities

- An annuity is a stream of equal cash flows that occur at regular intervals for a finite number of T periods.



- $PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \ldots + \frac{C}{(1+r)^T} = \sum_{i=1}^{T} \frac{C}{(1+r)^i} = \frac{C}{r}\left(1 - \frac{1}{(1+r)^T}\right)$

- We find C/r, just like in the perpetuity formula! We're just adjusting for time being finite

Pricing securities: Annuities

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- $C = 6000$, $r = 0.1$, $T = 30$. $PV = \frac{6000}{0.1}\left(1 - \frac{1}{1.1^{30}}\right) = 56561,48$

Pricing securities: Annuities

- You started a new company with seed money from your parents and friends of 18000. You expect your company to earn 1,000 (at the end of) each year for the next 4 years (beginning in year 1), but will then take off and grow 7% per year forever after that. The discount rate is 10%.

- Is this new company a good idea?

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- $PV = PV_1 + PV_2^{t=0} = 3169.87 + 24360.81 = 27530.68$

Pricing security: the value of a government bond

- French OAT(4Y) currently pay a rate of 2.5%: it pays interest every year + principal the last year.

- You invest 100 euros in year 0, in year 1 to 4 you get 2.5 euros and 100 euros more in year 4.

- Assume the discount rate is 2%.

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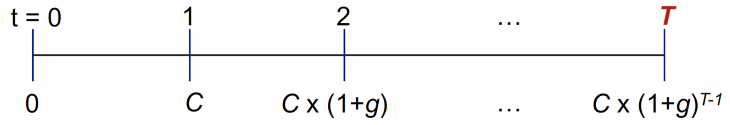
- The bond is a combination between an annuity and a final payment.
- $PV = \frac{2.5}{0.02} \times (1 - \frac{1}{1.02^4}) + \frac{100}{1.02^4} = 101.9$
- Bond prices decreases with r.

Pricing securities: Growing Annuities

- A growing annuity is like a regular annuity, except that the cash flow C grows at a constant rate g.



- $PV = \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + .. + \frac{C(1+g)^{T-1}}{(1+r)^T}. = \frac{C}{r-g}\left(1 - \frac{(1+g)^T}{(1+r)^T}\right)$

- In all these formulas, CF begins in period 1 (one period after the present). If there is a CF in period 0, just add it separately.

Calculating returns

- Here, we will look at different ways of compounding and quoting interest rates.
- The quoted rate is given on the basis that is customary in the text. We generally hear about yearly rates.
- However, rates can be paid at different intervals. What about if the yearly rate is paid daily? semi-annually?

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- However, rates can be paid at different intervals. What about if the yearly rate is paid daily? semi-annually?
- It will affect the future value differently. As a rule, any annual rate paid on a shorter basis will be value more.
  - Annual quoted rate r with annual compounding means in t years: $FV = PV(1+r)^t$
  - Annual quoted rate r with semi-annual compounding means in t years: $FV = PV(1+r/2)^{2t}$
  - Annual quoted rate r with daily compounding means in t years: $FV = PV(1+r/365)^{365t}$

The Effective Annual Rate (EAR)

- It is convenient to express compounded interest on an equivalent annual basis.
- The EAR is the annually compounded rate that would result on an equivalent future value.
- For a rate compounded m times a year:

$$PV \times \left(1 + \frac{quoted\ rate}{m}\right)^m = PV \times (1 + EAR) \Leftrightarrow EAR = \left(1 + \frac{quoted\ rate}{m}\right)^m - 1$$

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- What is the EAR of a 10% rate paid twice a year?
- $EAR = (1 + 0.1/2)^2 - 1 = 0.1025$ or 10.25%.

Continuous Compounding

- What happens to the FV when the rate is compounded continuously?
- Interesting theoretically and for professionals.
- $FV = PV \times e^{rt}$: simple formula.
- $EAR = e^{r} - 1$
- Suppose $r = 5\%$, $PV = 1000$, $t = 1$ year, what's the FV with annual compounding? With continuous compounding?

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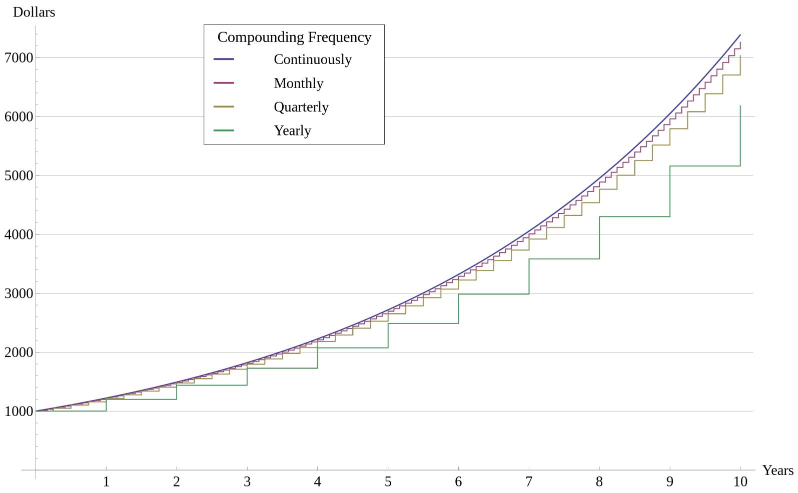
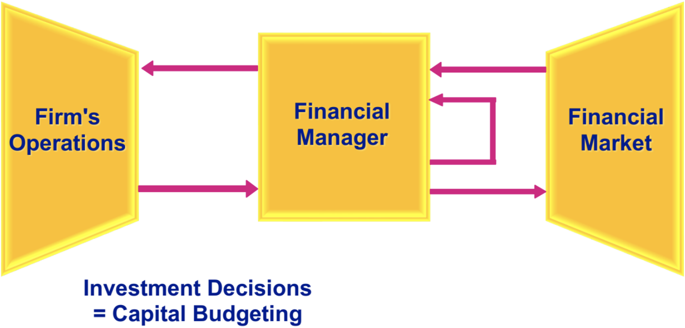
- $EAR = e^{r} - 1$
- Suppose $r = 5\%$, $PV = 1000$, $t = 1$ year, what's the FV with annual compounding? With continuous compounding?
- Periodic: $FV = 1000 \times (1.05)^{1} = 1050$.
- Continuous: $FV = 1000 \times e^{0.05} = 1051.27$.

Calculating returns



Investment decision rules



- How do firms decide what projects to invest in?
  - Facebook buys WhatsApp for $19bn (2014).
  - Elon Musk buys Twitter for $44bn (2022).
  - etc.

Investment decision rules

- We know how to compute the value of a project.
- Now, given the value of a project, is it worth investing in?
- We will study three decision tools:
    - The Net Present Value (NPV)
    - The Internal Rate of Return (IRR)
    - The Payback rule

The Net Present Value

- Reminder: The PV is the sum of the PV of individual CF:

$$PV = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + ... + \frac{C_T}{(1+r)^T}$$

- The NPV is computed when there are some costs: investment costs for instance.
- The NPV corresponds to the difference between the PV of benefits and the PV of costs.
- NPV decision rule: Invest if and only if NPV$>$0
- Careful: the NPV is sensitive to its assumptions: CF and r are uncertain.
  - Need to conduct a sensitivity analysis.
- First proposed as a systematic rule by Fischer (1907) but enters the theory of capital budgeting later in 1951.

The Net Present Value

Parameter	Initial Assumption	Worst Case	Best Case
Units Sold (thousands)	50	35	65
Sale Price ($/unit)	260	240	280
Cost of Goods ($/unit)	110	120	100
NWC ($ thousands)	1125	1525	725
Cost of Capital	12%	15%	10%

- Net working capital is a cash outflow ($\approx$ inventory)
- CF = (Sale Price - COGS)*units - NWC.

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- NPV is
  - Initial: $\frac{(260-110)*50-1125}{1+0.12} = 5692$
  - Worst: $\frac{(240-120)*35-1525}{1+0.15} = 2326$
  - Best: $\frac{(280-100)*65-725}{1+0.10} = 9977$

The Internal Rate of Return

- Suppose you have the opportunity to pay 100 today and receive 120 in one year, knowing that similarly risky investment offer a return of 10%. Should you do it ?
- We can directly see that the rate of return of this opportunity is 20%, which is larger than the rate of return of similar investments.
- Less simple to compute the rate of return when there are multiple periods and CF.
- How to generalize this concept?

The Internal Rate of Return

- It is useful to see that the NPV is a function of the discount rate:

$$NPV(r) = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + ... + \frac{C_T}{(1+r)^T}$$

- The IRR is the discount rate at which the NPV is exactly 0.

- It is the unknown in:

$$NPV(IRR) = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + ... + \frac{C_T}{(1+r)^T} = 0$$

- IRR is not the cost of capital but the hypothetical cost of capital that would make the NPV zero. IRR depends on the CF while *r* depends on the risk of the project.

The Internal Rate of Return

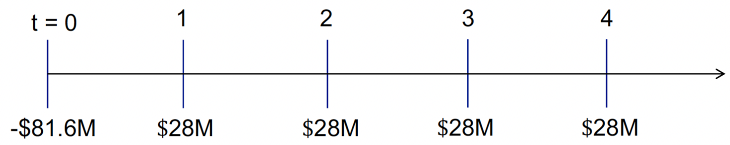
- What is the IRR in the two period case with C_0 the initial investment (< 0) and C_1 the unique CF?

The Internal Rate of Return

- What is the IRR in the two period case with C_0 the initial investment (< 0) and C_1 the unique CF?
- $NPV(IRR) = -C_0 + \frac{C_1}{1+IRR} = 0$
- $\iff IRR = \frac{C_1 - C_0}{C_0}$
- $\longrightarrow$ Profit divided by investment.

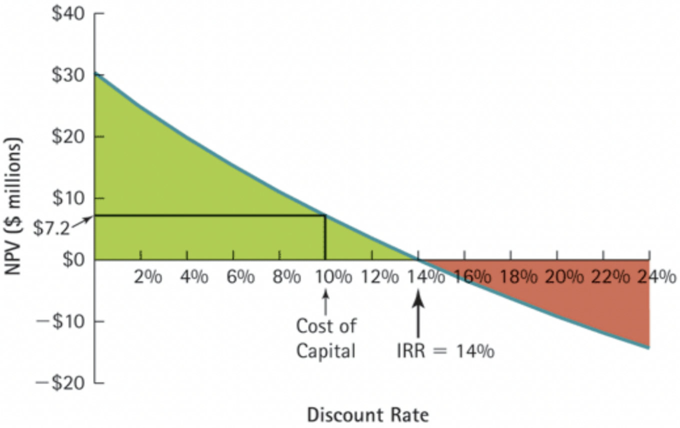
The Internal Rate of Return

- Generally, the IRR is not easy to determine.



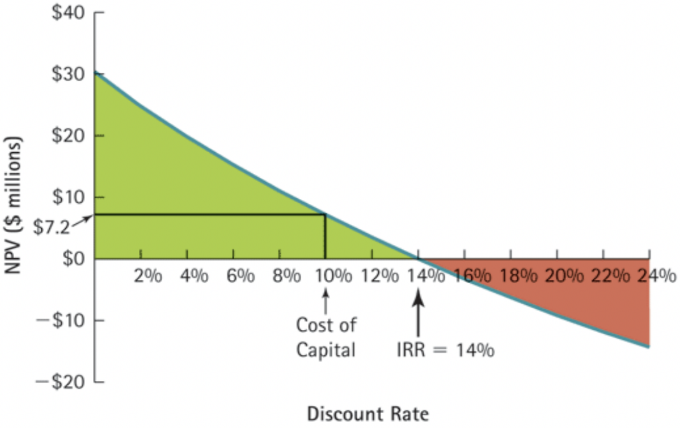
- By trial and error we can find it: compute $-81.6 + \frac{28}{R}\left(1 - \frac{1}{(1+R)^4}\right)$
- We obtain 0 for $R = 14$.

The Internal Rate of Return



The Internal Rate of Return

Naive Investment Rule



- Invest as long as the IRR is larger than the project cost of capital (the return of other alternatives with equivalent risk and maturity).

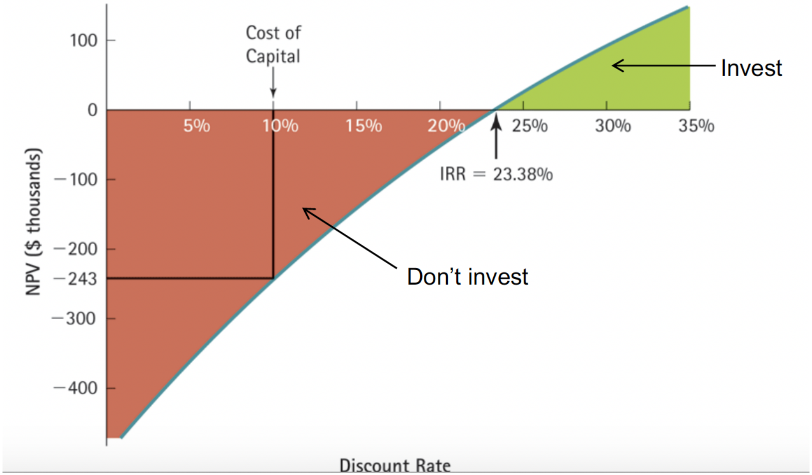
The Internal Rate of Return

Naive Investment Rule

- Why is it naive?
- It only works when the NPV is decreasing with the discount rate as in the case of investment projects.
  - When $C_0 < 0$ and next CF > 0.
- NPV can increase with the discount rate.
  - For debt for instance, where $C_0 > 0$ and other CF < 0.
- A less naive rule depends on the slope of $NPV(r)$.

The Internal Rate of Return

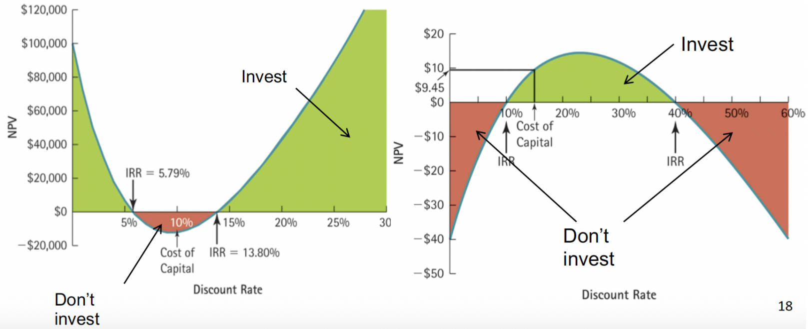
Naive Investment Rule



The Internal Rate of Return

Naive Investment Rule

- Still, it does not always work.
- $NPV(r)$ might not be a monotonic function.



The Internal Rate of Return

IRR or NPV?

- For NPV, you need to compute only one number: the NPV at the cost of capital r.
- For IRR, you need to know the entire NPV function, which can be a complicated one.
- IRR can be relevant for projects with a simple CF structure.

The Internal Rate of Return

Manipulating IRR

- Apollo, one of the world's largest asset manager reported an annual gross IRR of 39% over the past 30 years.

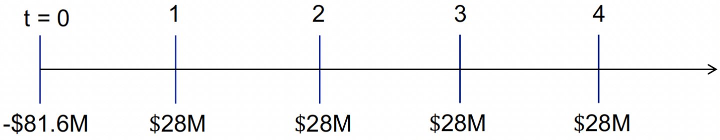
- Over the same period the SP500 return is 9%.

- But... a \$1bn investment of 39% for 30 years is worth 20tn, which is the GDP of the U.S. $\longrightarrow$ IRR is misleading!

- Consider the following project: $C_0 = -100$, $C_1 = 125$, $C_2 = 0, ..., C_5 = 0$.
    - Project IRR is $\frac{125}{(1+IRR)} = 100 \iff IRR = 25\%$.
    - But the annual return is: $R = \frac{125+0+0+0+0}{100}^{1/5} - 1 = 4.56\%$.
    - The IRR does not take into account that you earned 0 during 4 years.

Payback rule

- Payback is the amount of time it takes a project to pay back investment.
- Rule: accept the project if the payback is lower than a pre-specified amount of time.



- What is the payback period?

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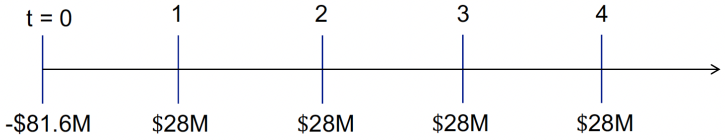

- What is the payback period?
- $28 \times 2 = 56 < 81.6 < 28 * 3 = 84$. It's 3 years.

Payback rule

Drawbacks

- It ignores the time value of money. An upgrade would be to compute a discounted payback period.
- It ignores CF after the payback period: the payback rule can suggest to invest even if the NPV is negative.
- How to chose the acceptable ¡number of years ?
- It is useful in case of an exogenous time constraint like a license to operate, a franchise agreement, etc.

Payback rule

Drawbacks

Project	C_0	C_1	C_2	C_3
A	−2,000	500	500	5,000
B	−2,000	500	1,800	0
C	−2,000	1,800	500	0

- Suppose the cutoff period is 2 years and $r = 10\%$

Payback rule

Drawbacks

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Payback rule

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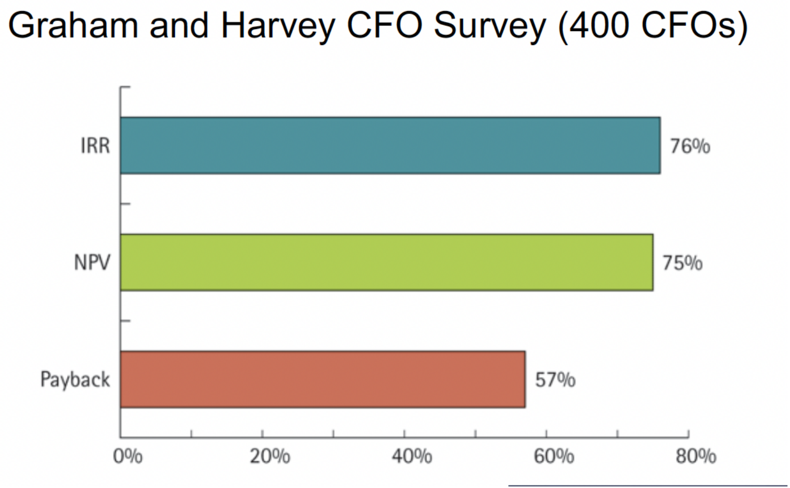
- Suppose the cutoff period is 2 years and $r = 10\%$
- Payback periods: 3, 2, 2: accept B and C
- NPV: 2624,-58,50: accept A and C

Choosing between projects in practice

- What do CFO do?
- Choosing among mutually exclusive projects.
- Choosing a project when resources are limited.
- Factors affecting the decision.

Choosing between projects in practice

What do CFO do?



Choosing between projects in practice

What do CFO do?

- NPV is the most informative and reliable rule.
- But it requires a lot of information:
  - Estimates of future CF.
  - Estimates of the cost of capital r.
  - Sensitivity analyses.
- Other rules can be simpler:
  - IRR doesn't require an estimate of r even if r is still needed to take the decision.
  - Payback rule does not require r nor CF far into the future (stop once the project is paid back).
- The practical decision must trade off accuracy and complexity.
- A good practice is to use several different rules.

Choosing between projects in practice

Choosing among mutually exclusive projects.

- Choose the project with the larger NPV
- It does not mean with the larger IRR!

	Investment	CF 1st year	Growth rate (g)	Cost of capital (r)
1. Bar	$400,000	$60,000	3.5%	12%
2. Coffee Shop	$200,000	$40,000	3%	10%
3. Apparel Store	$500,000	$75,000	3%	13%

Choosing between projects in practice

Choosing among mutually exclusive projects.

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- $NPV = \frac{CF}{r-g} - C_0$
- $\frac{CF}{IRR-g} - C_0 = 0 \iff IRR = \frac{CF}{C_0} + g$

Choosing between projects in practice

Choosing among mutually exclusive projects.

The cost of capital (r) is 12%, and the expected CF streams are:

t = 0	1	2	3
(1) -$10,000	$5,000	$5,000	$5,000
(2) -$10,000	$6,000	$6,000	$6,000
(3) -$50,000	$25,000	$25,000	$25,000

Choosing between projects in practice

Choosing among mutually exclusive projects.