

## Correction of the exercise 4 of PS 1

**There are two periods in this problem: now ( $t=0$ ) and tomorrow ( $t=1$ ). Assume an individual has 1000\$ now. He can invest in machines, which will produce, without risk, a return of 20%.**

- a. How much will he get in the next period if he invests everything in machines? If he only invests 750\$?**

If he invests everything (\$1000):

- Return tomorrow =  $\$1000 \times (1 + 0.20) = \$1000 \times 1.20 = \$1200$

If he invests only \$750:

- Return tomorrow =  $\$750 \times 1.20 = \$900$
- (The remaining \$250 stays as cash, no return)

**b. The interest rate in the market is 7%. Instead of investing he can lend his money and benefit from the interest during the next period. How much will he get in the next period if he lends everything? If he invests 750\$ and lends 250\$?**

If he lends everything (\$1,000):

- Return at  $t=1$  =  $\$1000 \times (1 + 0.07) = \$1000 \times 1.07 = \$1070$

If he invests \$750 in machines and lends \$250:

- From machines:  $\$750 \times 1.20 = \$900$
- From lending:  $\$250 \times 1.07 = \$267.50$
- Total at  $t=1$  =  $\$1167.50$

**c. The bank agrees to lend him some funds based on what he will have next period. But the bank thinks that the investment is risky so that it only agrees to apply a tax rate of 10%.**

- i. How do we call the difference between the interest rate in the market and the rate he gets from the bank?**

The difference between the market interest rate (7%) and the rate the bank applies (10%) is called the credit spread or risk premium. This spread compensates the bank for the perceived risk of the investment.

- ii. How much can he borrow if he invests 750\$? 1000?**

The bank lends at 10% interest, meaning the individual must repay 1.10 times what he borrows. We use the formula  $PV = \frac{FV}{(1+r)^t}$

If he invests \$750:

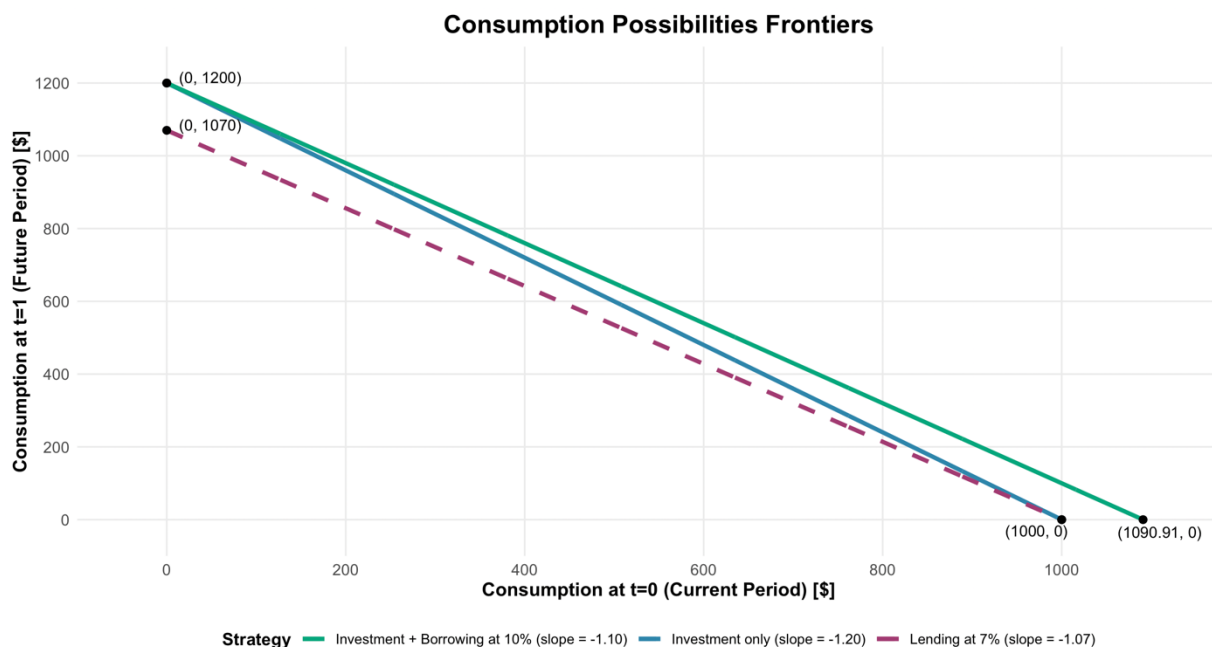
- Value at  $t=1$  from \$750 investment =  $\$750 \times 1.20 = \$900$

Maximum borrowing satisfies:  $PV = \frac{FV}{(1+r)^t}$  i.e.  $900 / 1.10 = \$818.18$

If he invests \$1000:

- Value at  $t=1$  from \$1000 investment =  $\$1000 \times 1.20 = \$1,200$
- Maximum borrowing B satisfies:  $PV = \frac{FV}{(1+r)^t}$  i.e.  $1200 / 1.10 = \$1090.91$

c. Draw a diagram with all these possibilities. Conclude on the most profitable strategy.



The most profitable strategy consists in investing and borrowing against his future revenue in period 1. This allows him to extend its consumption possibility frontier *when his investment is successful*.