Problem Set 4: Risk and CAPM

Correction of Questions 5 and 7

Use the Course 5 CAPM and asset-beta formulas. Final rates are rounded to two decimals; intermediate values are retained without rounding.

Question 5: Electric-car comparables

Your firm is considering expanding into the electric-car business. You have the following information on other firms in the industry:

	Equity beta	Stock price	Number of shares	Debt
Tesla (electric only)	2.07	\$242	3.17 billion	\$5.81 billion
Rivian (electric only)	2.29	\$21	931.51 million	\$3.22 billion
Ford (electric and non-electric)	1.61	\$12	3.93 billion	\$144.85 billion

The risk-free rate is 5% and the historical market risk premium is 4%. There are no taxes. Assume debt betas are zero and firms maintain a constant debt-to-value ratio. What is the cost of equity for an unlevered firm (100% equity financed) in the electric-car business?

Answer. Use **Tesla and Rivian** as pure-play comparables. Ford also produces non-electric vehicles, so its beta does not isolate the electric-car business.

1. Compute the market value of equity. Use $E = \text{Stock price} \times \text{Number of shares}$, with all amounts below in USD billions:

$$E_T = 242 \times 3.17 = 767.14, \qquad E_R = 21 \times 0.93151 = 19.56171.$$

2. Remove the effect of debt. The course's asset-beta formula is

$$\beta_A = \frac{E}{E+D}\beta_E + \frac{D}{E+D}\beta_D.$$

Since $\beta_D = 0$,

$$\beta_{A,T} = \frac{767.14}{767.14 + 5.81} \times 2.07 = 2.054440520,$$
$$\beta_{A,R} = \frac{19.56171}{19.56171 + 3.22} \times 2.29 = 1.966328072.$$

Following the course's comparable-company method, take the simple average:

$$\overline{\beta}_A = \frac{2.054440520 + 1.966328072}{2} = 2.010384296.$$

3. Apply CAPM. For the unlevered firm, equity finances all assets, so $\beta_E = \beta_A$. Therefore

$$r_E = r_f + \beta_E(r_M - r_f) = 5\% + 2.010384296 \times 4\% = 13.04153718\% \approx \boxed{13.04\%}.$$

Question 7: ZERO's divisional costs of capital

ZERO invests in gold and cars. The financial manager wants to estimate ZERO's cost of capital. The equity risk premium is 9.5% and the risk-free rate is -1%. Gold is countercyclical, so the average beta for a gold company is -0.1. Three similar car companies have betas of 1.5, 2.0, and 2.5. ZERO is financed only with equity, and 25% of its business is cars.

For the computations, interpret the business shares as market-value weights. With no debt, the company cost of capital equals its cost of equity. Use

$$r_i = r_f + \beta_i(r_M - r_f), \qquad \beta_{\text{portfolio}} = \sum_i w_i \beta_i.$$

The 9.5% is already the *market risk premium* $r_M - r_f$.

a. *What is ZERO's cost of capital for the gold division?*

Answer.

$$r_G = -1\% + (-0.1) \times 9.5\% = \boxed{-1.95\%}.$$

Gold's negative beta gives it a negative risk premium: its countercyclical returns provide insurance against market risk.

b. *What is ZERO's cost of capital for the cars division?*

Answer. Average the comparable car-company betas, then apply CAPM:

$$\beta_C = \frac{1.5 + 2.0 + 2.5}{3} = 2.0, \qquad r_C = -1\% + 2.0 \times 9.5\% = \boxed{18.00\%}.$$

c. *What is ZERO's total cost of equity capital?*

Answer. Gold has weight 75% and cars 25%. The firm's beta and cost of equity are

$$\beta_{\text{ZERO}} = 0.75(-0.1) + 0.25(2.0) = 0.425,$$
$$r_{E,\text{ZERO}} = -1\% + 0.425 \times 9.5\% = 3.0375\% \approx \boxed{3.04\%}.$$

Equivalently, $r_{E,\text{ZERO}} = 0.75(-1.95\%) + 0.25(18.00\%)$.

ZERO then decides to open a hydrogen-car company alongside its gold investment. Half the business is gold and half produces hydrogen cars.

d. *If the expected return of the combined company is 10%, what must the beta of the hydrogen-car industry be? Assume no debt.*

Answer. Under CAPM, the stated expected return equals the required return. First recover the combined beta, then use the new 50/50 weights:

$$\beta_{\text{combined}} = \frac{r_{\text{combined}} - r_f}{r_M - r_f} = \frac{0.10 - (-0.01)}{0.095} = 1.157894737,$$
$$1.157894737 = 0.50(-0.1) + 0.50\beta_H,$$
$$\beta_H = \frac{1.157894737 + 0.05}{0.50} = 2.415789474 \approx \boxed{2.42}.$$

Check: $r_H = -1\% + 2.415789474 \times 9.5\% = 21.95\%$, and $0.50(-1.95\%) + 0.50(21.95\%) = 10.00\%$.