Introduction to Finance

Problem Set 4

Question 1

Here are the returns and standard deviations for four investments.

Security	Return (%)	Standard deviation (%)
Treasury bills	6	0
Stock P	10	14
Stock Q	14.5	28
Stock R	21	26

Calculate the standard deviations of the following portfolios.

- a. 50% in Treasury bills and 50% in stock P.
- b. 50% each in Q and R, assuming the shares have (i) perfect positive correlation, (ii) perfect negative correlation, and (iii) no correlation.

Question 2

Mark Harrywitz proposes to invest in two shares, X and Y. He expects returns of 12% from X and 8% from Y. The standard deviations are 8% for X and 5% for Y, and the correlation between the returns is 0.2.

- a. Compute the expected return and standard deviation of the following portfolios: 50% X / 50% Y; 25% X / 75% Y; and 75% X / 25% Y.
- b. Sketch the set of portfolios composed of X and Y.
- c. Suppose Mr. Harrywitz can also borrow or lend at 5%. Show on your sketch how this alters his opportunities. Given borrowing or lending, what proportions of the common-stock portfolio should be invested in X and Y?

Question 3

ChemCo has an 8% debt cost of capital and a 15% equity cost of capital. Its debt has a market value of $500 million in perpetual bonds with a promised yield of 10%. There are 10 million shares outstanding, each worth $50. The risk-free rate is 4% and the expected market return is 12%. Calculate:

- a. Equity beta and debt beta.
- b. Return on assets and asset beta.

Question 4

The liability side of Cox Chemicals' balance sheet has the following components:

Security	Beta	Market value (USD millions)
Supersenior debt	0	28
Senior debt	0.1	250
Junior debt	0.15	75
Equity	0.9	340

All values are market values. The risk-free rate is 5% and the expected market return is 11%. Assume the CAPM holds. Ignore taxes, transaction costs and financial-distress costs. Initially, Cox holds no excess cash.

- a. Use the CAPM to calculate the expected return on each of the four securities.
- b. Compute Cox's WACC directly as the market-value-weighted average of the required returns on the four securities. Treat each debt tranche separately.
- c. Compute Cox's asset beta as the market-value-weighted average of the four security betas. Use this average beta and the CAPM to calculate the required return on Cox's assets. Compare your answer with the WACC in part b and explain why the two methods give the same result.
- d. Prove the general result for two assets A and B with market-value weights ω_A and ω_B, where $\omega_A + \omega_B = 1$, expected returns r_A and r_B, and betas β_A and β_B. Assume both assets satisfy the CAPM with the same risk-free rate r_f and expected market return r_M. Show that the WACC computed as the weighted average of their CAPM returns equals the CAPM return computed using the weighted average beta $\bar{\beta} = \omega_A \beta_A + \omega_B \beta_B$.

Question 5

Your firm is considering expanding into the electric-car business. You have the following information on other firms in the industry:

	Equity beta	Stock price	Number of shares	Debt
Tesla (electric only)	2.07	$242	3.17 billion	$5.81 billion
Rivian (electric only)	2.29	$21	931.51 million	$3.22 billion
Ford (electric and non-electric)	1.61	$12	3.93 billion	$144.85 billion

The risk-free rate is 5% and the historical market risk premium is 4%. There are no taxes. Assume debt betas are zero and firms maintain a constant debt-to-value ratio. What is the cost of equity for an unlevered firm (100% equity financed) in the electric-car business?

Question 6

XYZ Inc., a diversified conglomerate, is deciding whether to buy a copper mine. XYZ already owns gold mines and has recently invested in biotechnology. XYZ's current cost of capital is 10%. Market data for other companies appear below. Assume all debt betas equal zero.

Firm	Industry	Shares (millions)	Price/share	Debt (USD millions)	Equity beta
A	Gold/Biotech	3	$10	15	1
B	Copper	1	$5	1	1.02
C	Copper	2	$20	0	0.8
D	Copper	1.5	$3	3	1.37

What opportunity cost of capital should XYZ use to evaluate the copper mine? Use a risk-free rate of 7% and a market risk premium (rm − rf) of 8%.

Question 7

ZERO invests in gold and cars. The financial manager wants to estimate ZERO's cost of capital. The equity risk premium is 9.5% and the risk-free rate is −1%. Gold is countercyclical, so the average beta for a gold company is −0.1. Three similar car companies have betas of 1.5, 2.0, and 2.5. ZERO is financed only with equity, and 25% of its business is cars.

- a. What is ZERO's cost of capital for the gold division?
- b. What is ZERO's cost of capital for the cars division?
- c. What is ZERO's total cost of equity capital?

ZERO then decides to open a hydrogen-car company alongside its gold investment. Half the business is gold and half produces hydrogen cars.

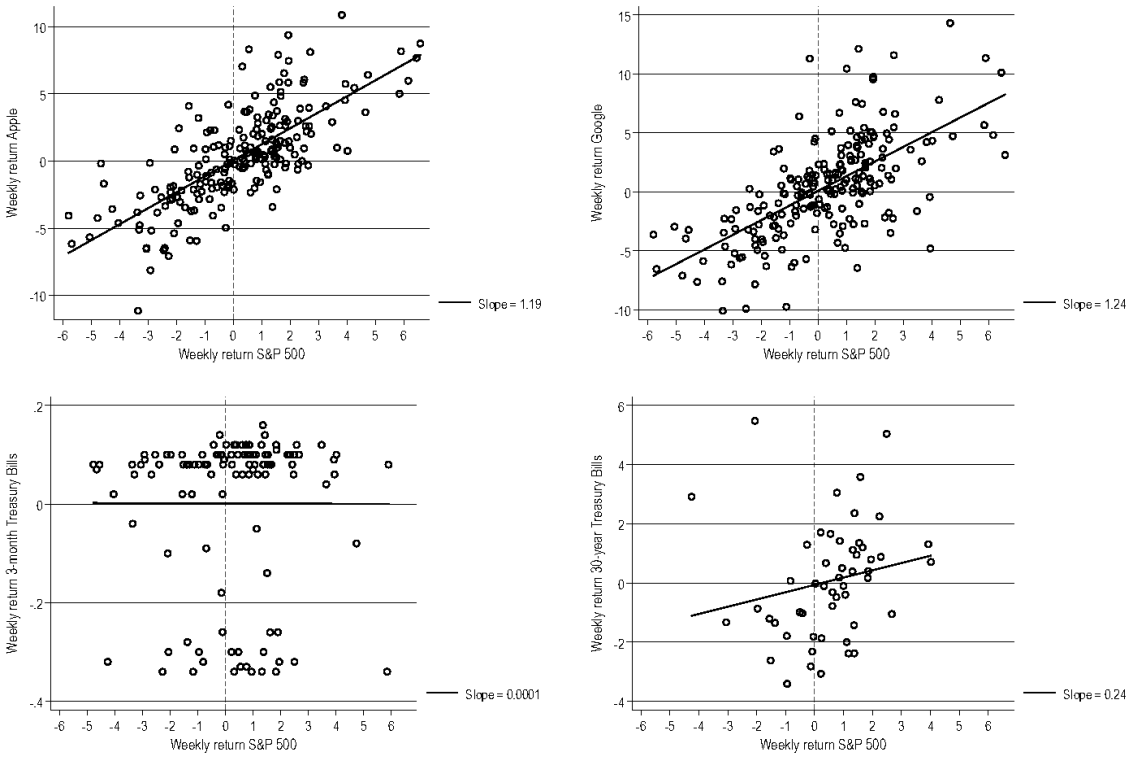
- d. If the expected return of the combined company is 10%, what must the beta of the hydrogen-car industry be? Assume no debt.

Question 8

Google is planning to build a phone. Its main competitor will be Apple.

Google has debt with a market value of $27 billion and a 5% cost of debt. Its stock price is $173 and it has 12,343,000,000 shares outstanding. The expected market return is 12% and the corporate tax rate is 21%. The return on 3-month Treasury bills is 4.5%; the return on 30-year Treasury bonds is 4.6%.

The four figures show weekly security returns against S&P 500 returns. Each fitted-line slope is given in its legend. Treat the Treasury yields as annual rates.



- b. Which asset can we assume is the risk-free asset? What is the beta of Google stock? What is the beta of Apple stock?
- c. Using the CAPM, determine Google's cost of equity. You do not need to unlever the beta.
- d. What is Google's weighted average cost of capital (WACC)?